Report on Problem 14: Looping Pendulum
GYPT – 2019

NAME

(Team: NAME)

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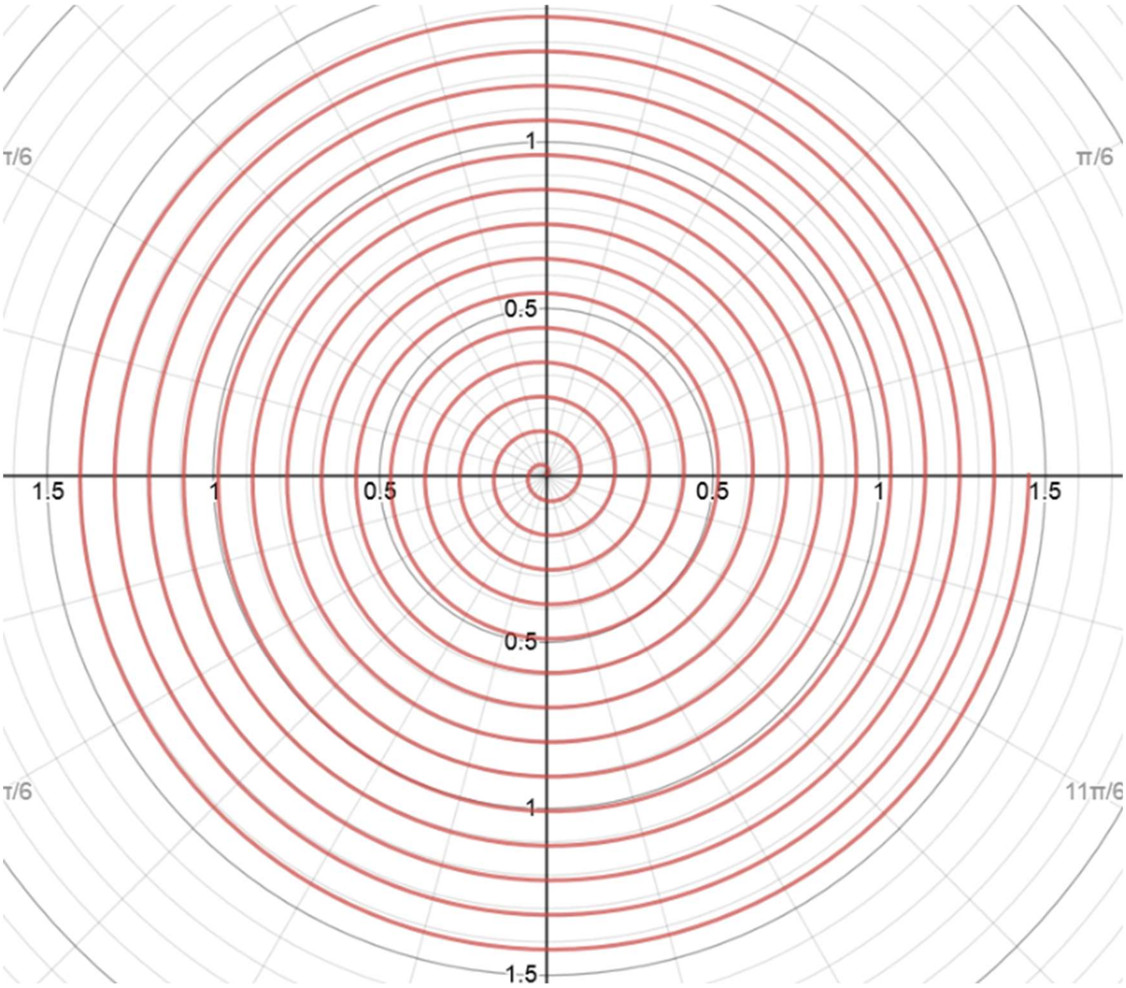
E-Mail



Contents

Introduction

In this report we will look at the following problem:

Connect two loads, one heavy and one light, with a string over a horizontal rod and lift up the heavy load by pulling down the light one. Release the light load and it will sweep around the rod, keeping the heavy load from falling to the ground. Investigate this phenomenon.

Our objective is to explain the phenomenon and to comprehend of which constraints it depends. We will orient ourselves along following guidelines/central questions:

1) We will conduct the experiment as shown in the video (source 1 – see bibliography) and change various parameters of the experiment systematically, in order to deduct their influence on the outcome of the experiment.

2) We want to create a theoretical model that takes into account the relevant parameters and can explain (and predict) outcomes of the experiment.

3) We want to evaluate how this model matches with our experimental data and explain possible discrepancies.

Basic explanation:

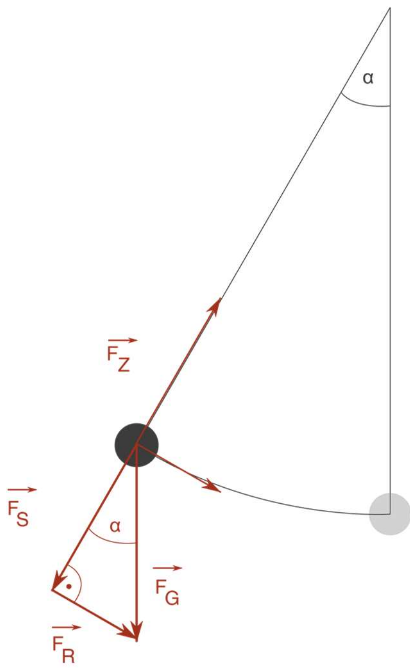
The light weight loops around the rod, performing a spiral movement. The rope wraps around the rod and the heavy weight falls, due to the gravitational force working upon it. Because of the spiral movement, a centripetal force is exerted on the light weight that, together with the friction of rod and rope acts against the gravitational force of the heavy weight. These counterforces stop the fall of the heavy weight.

Background research:

In a simple mathematical pendulum (see figure and compare source 2), there is a restorative force F_R. This force results from the tension of the rope, which acts as a centripetal force and the gravitational force F_G.
The period of oscillation for small angles is constant with $T=2\pi*\sqrt{(l/g)}$.
For larger angles of deflection the oscillation is not harmonic, but can be described through

$T(\varphi_{max}) = T(\alpha_{max}) = T_0/M(1,\cos(\varphi_{max}/2))$

where T_0 is the period for an infinitesimal amplitude/angle and M is the arithmetic-geometric mean (AGM). For the calculation of the latter an external website (source 3) can be used.



Own research

Theoretical Model

A_1 and A_2 are the starting points of the weights with the masses m_1 and m_2, respectively. After t_0 time the larger weight is stopped before reaching the ground - this always happens, otherwise (interpretation of the problem description) the experiment is considered a failure. Then, m_1 has moved from A_1 to E_1 and m_2 has moved from A_2 to E_2. The rope has length l, with $l=l_1+l_2$.
R is the amount of rope that has after t_0 been wrapped around the rod. And H is the falling height of the heavy weight. If the initial angle of deflection is 90°, i.e. the string is parallel to the ground, the vertical movement of m_2 is

$h=\sin(\alpha)*(l_2-H-R)$
In the following theory, the properties of the rope, except for the length, are neglected, although I consider friction and rigidity to be important factors as well. It is assumed that the rope is always stretched.
One gets the following energy transformatinon:

$\Delta E= m_2*g*(l_2-H-R)*\sin\alpha+m_1*H*g = 0{,}5*m_2 v_2^2$

This can be solved for the velocity

$v_2 = \sqrt{(2g*(l_2-H-R)* \sin\alpha + 2(m_1/m_2)*g*H)}$

In each instant, the movement of the light weight can be interpreted as that of a weight moving (approximately) in a circle with a centripetal force F_Z. Together with the friction, F_Z has to be equal or greater than the gravitational force F_{G1} in order to stop the heavy weight from falling:

$F_{G1} = m_1*g = m_2v_2^2/r +F_f = F_z +F_f = (2m_2g* \sin\alpha*(l_2-H-R)+2m_1gH)/r +F_f$

The radius r of the spiral is equal to l-H-R.

$F_{G1} = m_1*g = m_2v_2^2/r +F_f = F_z +F_f = 2m_2g* \sin\alpha+2m_1gH/(l_2-H-R) +F_f$
$2m_2* \sin\alpha+2m_1H/(l_2-H-R) + F_f /(g*(l_2-H-R) = m_1$

Since l-H-R gets smalller and smaller, according to this equation, the heavy weight is always stopped. However, the light weight at some point hits the rod and bounces back, the string can rip or the weights can collide.
Neglecting the force F_f of the friction again, I get:

$H = (l-R)*(2m_2*\sin\alpha-m_1)/(2m_2*\sin\alpha-3m_1)$

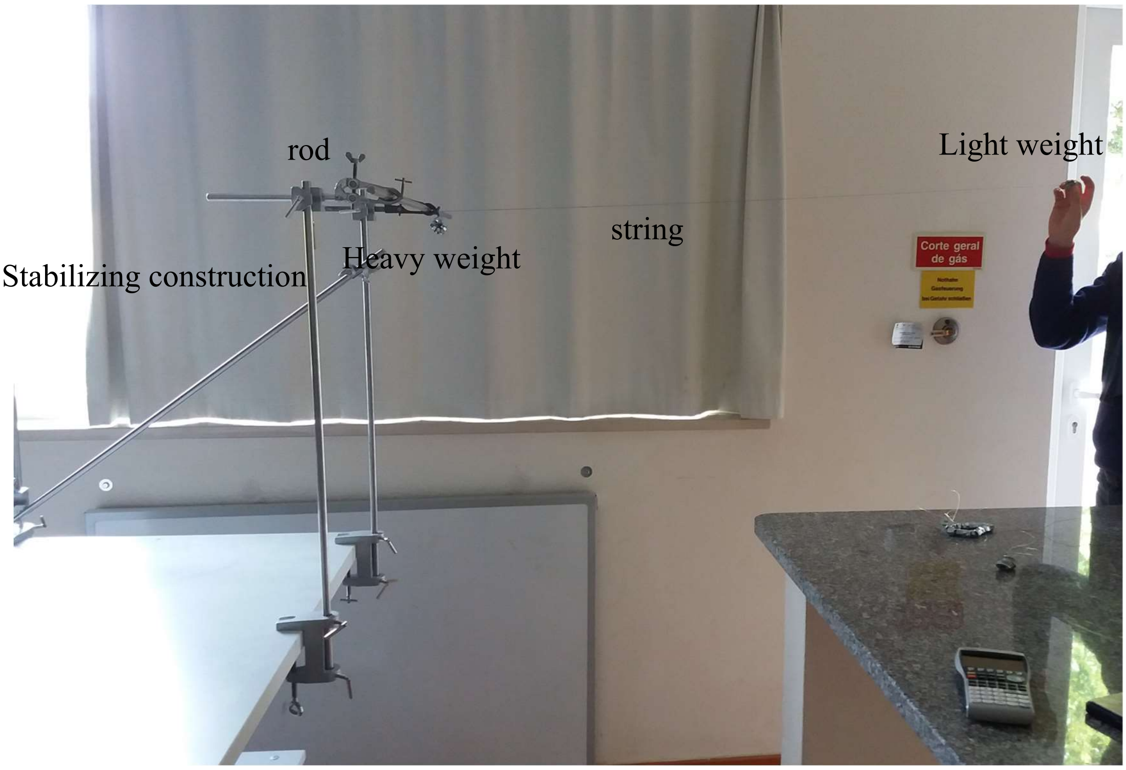
$\sin\alpha = (m_1*(l_2-H-R)-2m_1*H)/(2m_2*(l_2-H-R))$

The falling height H can be measured and R can be neglected in many cases

From this point onwards, the spiral is defined through the radius of the rod, because each lap around the rod must reduce the remaining l_1-H-R by $2*\pi*k$, where k is the radius of the rod. An open question is how much time each lap takes (→ work until GYPT).

Hypotheses, experiments, and explanations:

We start by building the following experimental setting:



Hypotheses:

The drop height and the form of the spiral depend upon

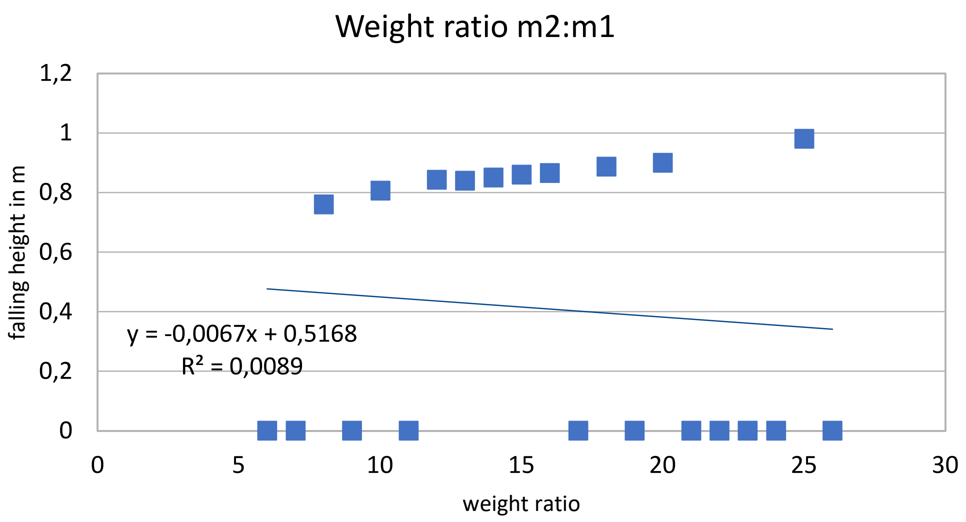
I) the ratio of the two weights
II) the length of the rope l_1 (See figure), but not l_2
III) the angle of deflection (which together with the length of the rope defines the amplitude/height)
IV) the radius of the rod
V) the rigidity and elasticity of the rope
VI) the friction of the rope with itself and the rod

Experiments and explanations:

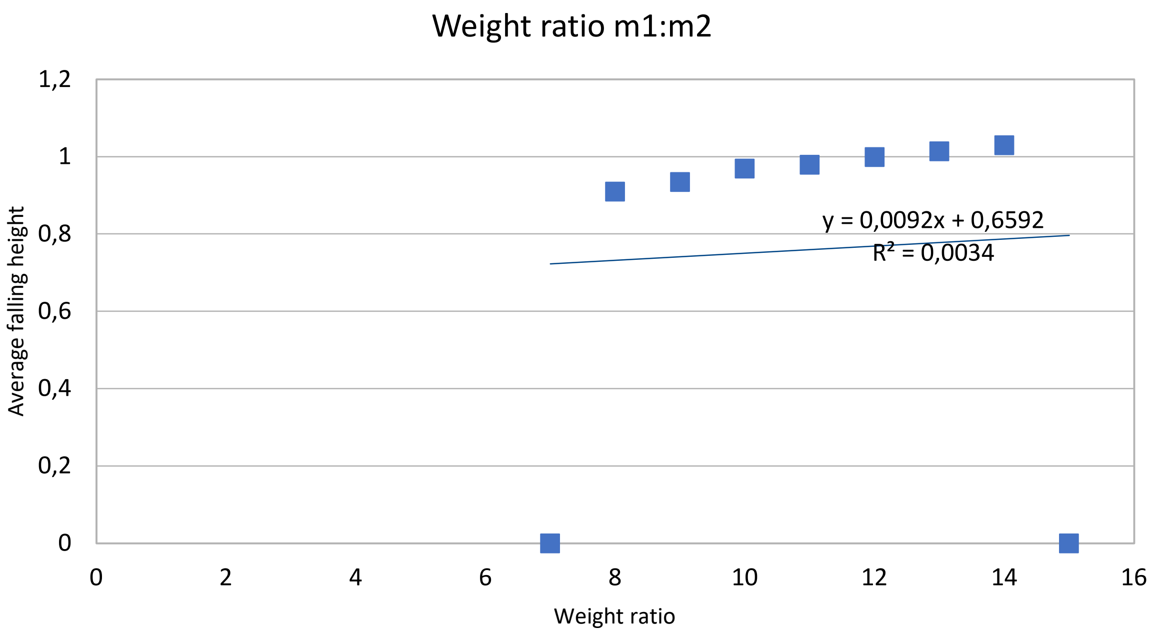
Regarding Hypothesis I:

The same string (properties and length) and the radius of the rod are kept, but the ratio of the weights is varied in two experiments with different constants. Then the drop distance of the heavy weight is measured three times and the average taken.

First we use a length l_1= 1,10m, angle= 90°; red rope; rod radius = 1,15cm:



Then we use a length l_1=1,40m; angle= 90°; blue rope; rod radius = 1,15cm:



Our measurements show that **if the ratio m_1:m_2 becomes larger, the drop distance increases proportionally.** We find a **linear relationship with an R^2 of over 0,96.** This finding fits in with reference 10. The experiment ceases to work, when the weight ratio is too big, because the velocity of the lighter weight gets too large and the string rips.

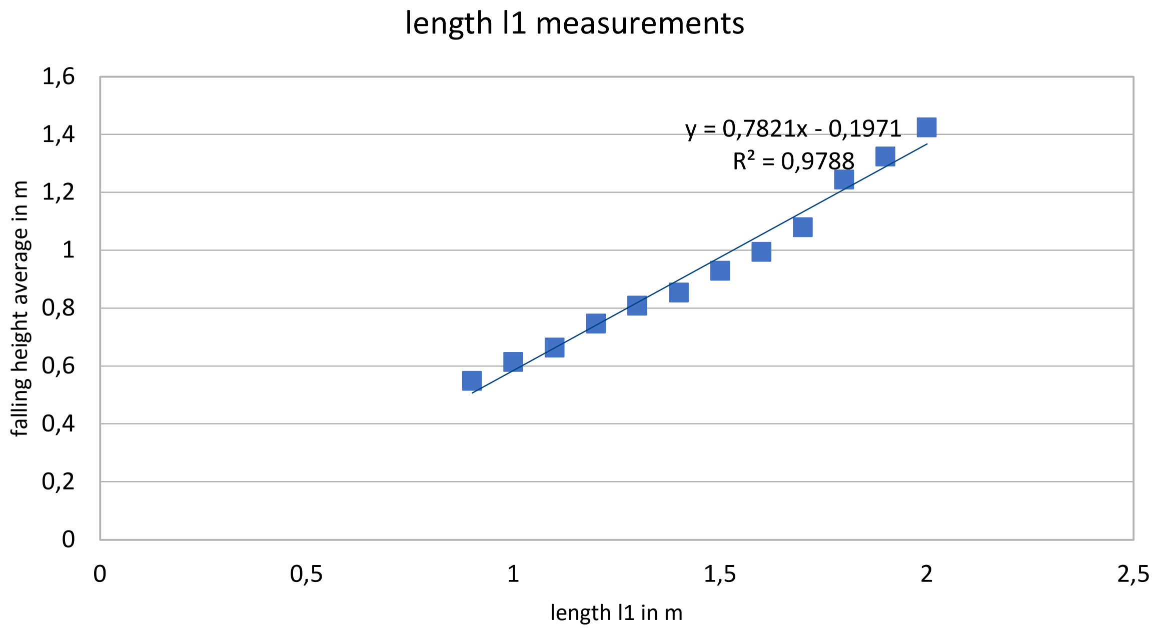
Remark: In may be the cas that an elastic rope will become more stretched, if larger weights are used. Therefore, it is not advisable for the measurement of the influence of the weight ratio to use such a rope, as it may lead to errors.

Regarding Hypothesis II)

It seems intuitive that l_2 is irrelevant, because the heavy weight is exerting a gravitational force which is independent of the height above the floor or l_2. However there are the following two qualifications: l_2 may not be such that the two weights collide and it may not

be so long that the height of the heavy weight above the ground is to small. An experiment, in which two strings are used, one short and one long,
and l_2 is varied, while all else (including l_1) remains equal, confirms this.The falling heights - final l_2 minus initial l_2 - are the same (apart from slight measurement errors).

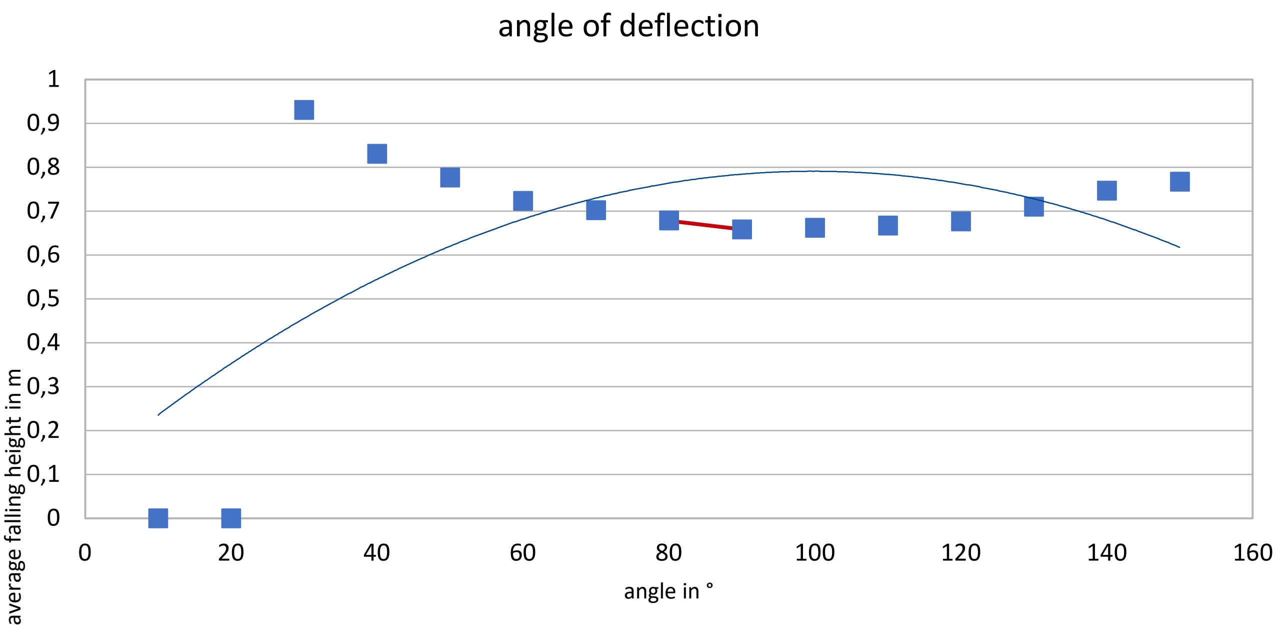
In order to measure the influence of l_1, it is varied, while keeping the weight ratio constant at 8:1, using the blue rope and a rod with radius 1,15 cm. (Because the same string is used, different initial l_2 have to be substracted.) The measurement indicates a linear relationship between l_1 and the falling height H:



When l_1 gets larger, the radius of the spiral increases and thus the centripetal force decreases, since we divide by l-H-R. And the increase in velocity due to an increase in l-H-R does not completely outweigh this, for $2(m_1/m_2)*g*H$ stays the same in the term $v_2{}^2 = 2g*(l-H-R)*\sin\alpha + 2(m_1/m_2)*g*H$. So the counterforce to F_{G1} decreases when l_1 gets bigger, which means that the heavy weight falls further until it is stopped by an increased friction and F_Z.

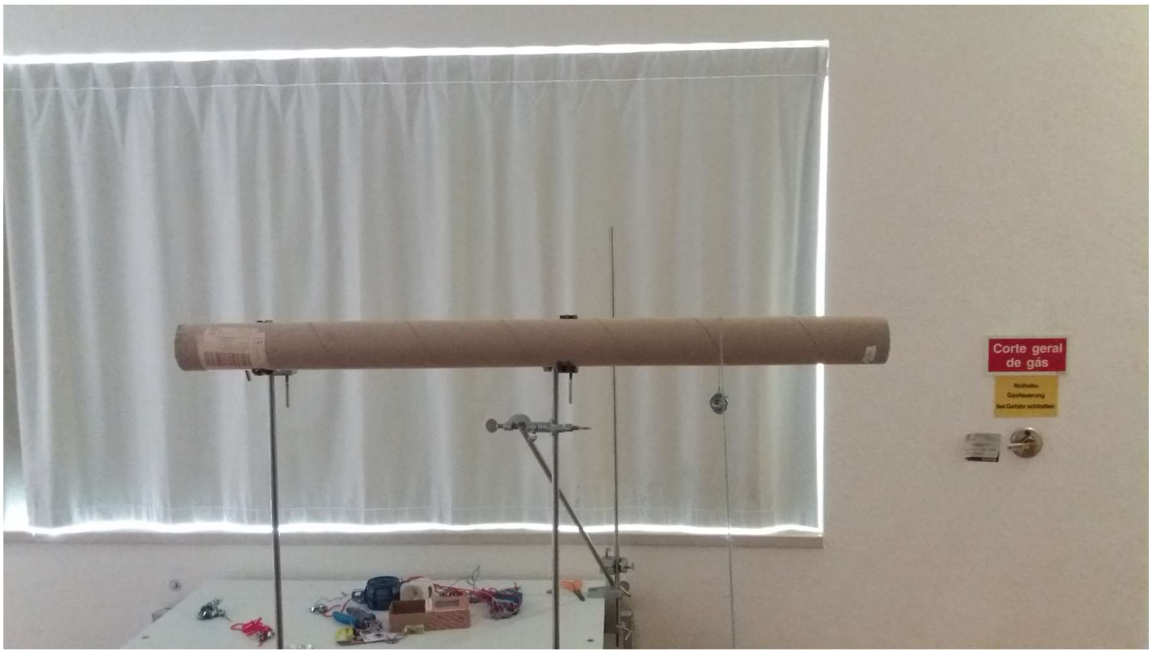
Hypothesis III):

We conduct an experiment with a weight ratio 8:1 ; l1=1,4 ; blue rope and change the angle of deflection:



The lowest average falling height measured is for an angle of 90° which we defined as the string being parallel to the ground (see sketch). Our theoretical model cannot yet explain this, because for h=sin(α)*(L-H-R) the angle of deflection needs to be 90°. I have not yet found a suitable alternative

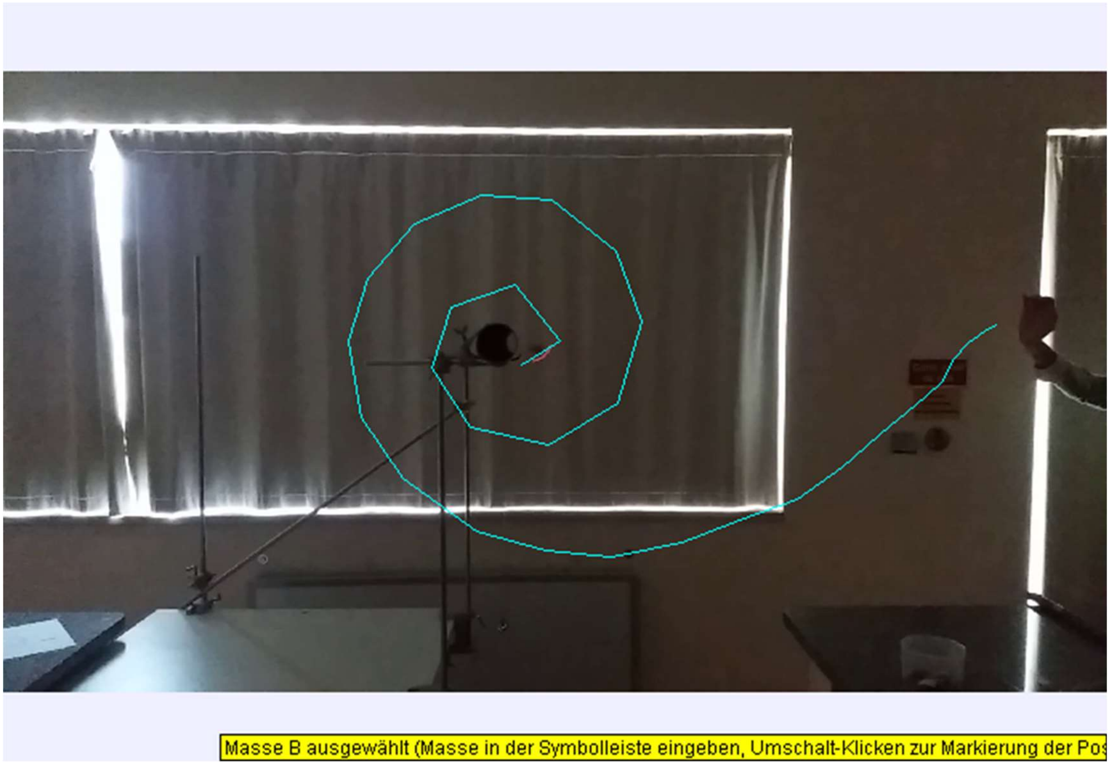
Hypothesis IV):
We use different rod radiuses, controlling for all else:



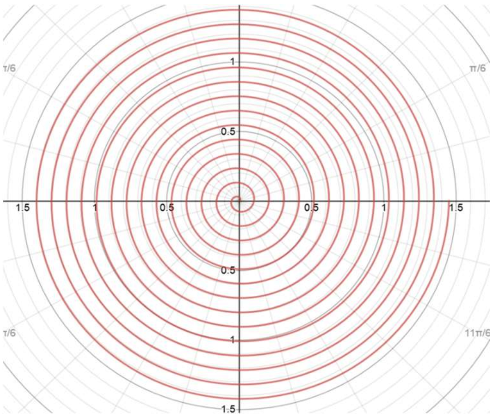
Remark: The sum of the drop height and the laps times the perimeter of the rod is a way to test the accuracy of the measurements. If it is significantly different from l_1 (measured beforehand), the measurement should be repeated.

First measurements indicate that the radius of the rod is of tremendous importance for the experiment and defines the number of laps performed around the rod by the light weight. Moreover, the radius of the rod defines the shape of the spiral, since for each lap the amount of rope that wraps around the spiral is equal to the perimeter of the rod (see fotos below).
The experimental data cannot yet be considered as sufficient to find a relationship (linear,exponential,etc?) between radius and laps/shape. Therefore, this part of the phenomenon requires further research (→ work until GYPT final). My conjecture is that the realtionship is linear, as the perimeter of the rod is equal to $2\pi * k$.

Radius 3,7 cm



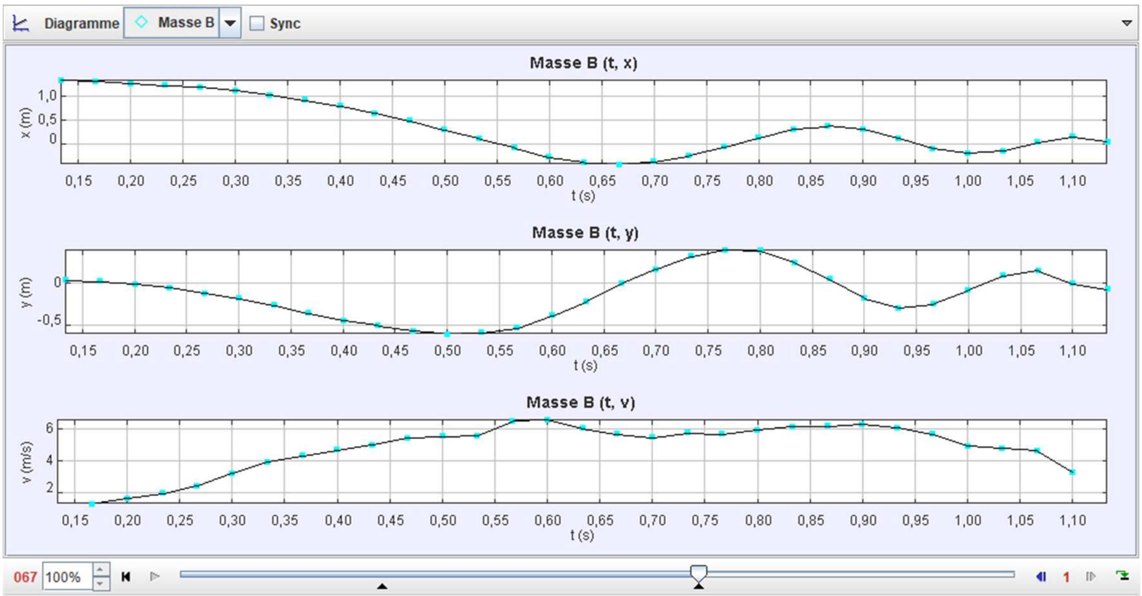
Radius 0,55 cm (some points were added manually due to camera issues)



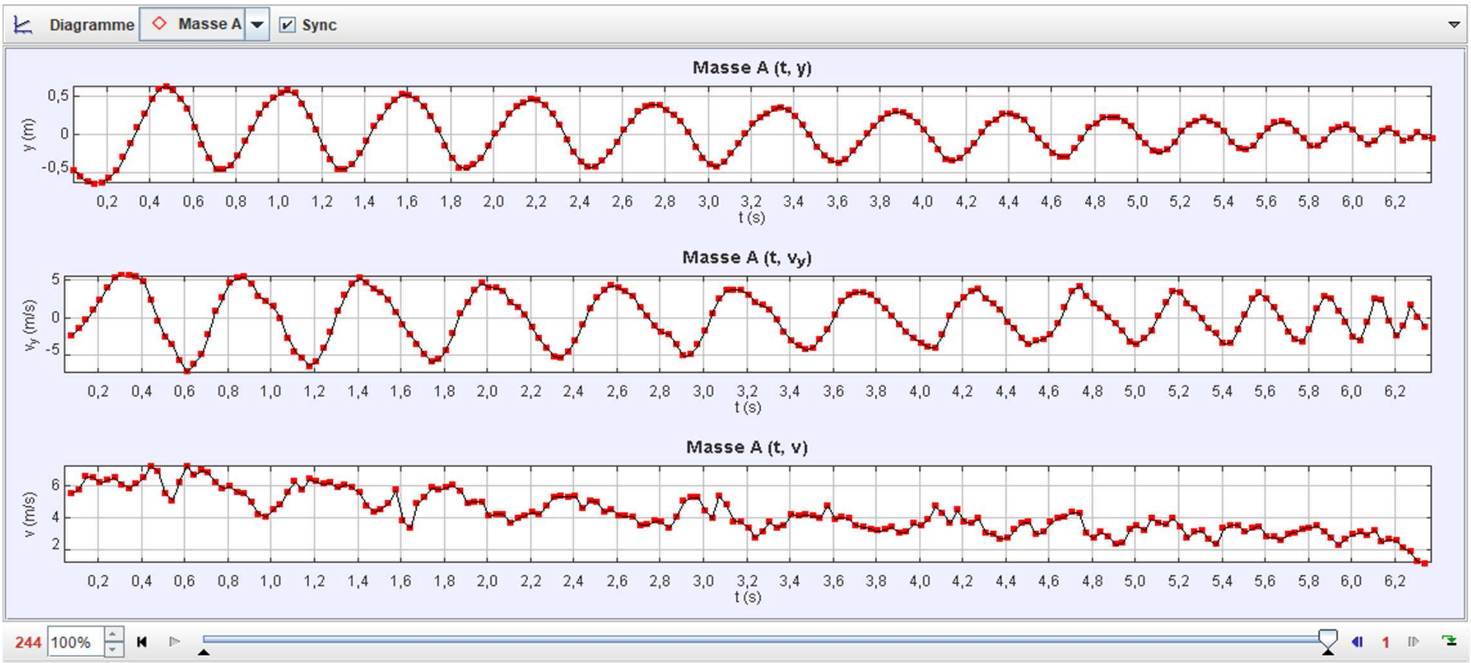
Archimedes spiral with 14 laps
as comparison (source 16)

The fotos below indicate that there is a periodical movement in X- and Y-direction, like an Archimedes Spiral. However, the first lap does not quite fit in with a perfect spiral. The velocity of the light weight‘s veloctiy increases when its potential energy is transformed into kinetic energy. The velocity decreases when this energy is retransformed into potential energy. It decreases towards the end because of energy losses through friction. As the spiral (X- and Y- movements) is periodic, the X- and Y- velocites and accelerations are too. The absolute value of the velocity has a less “smooth“/periodic graph, but should be too (there probably are some measurement errors):

Radius 3,7 cm, Movement in X-direction, Movement in Y-direction and total velocity:



Radius 0,55 cm, X- Movement, Y-Movement and Absolute velocity:



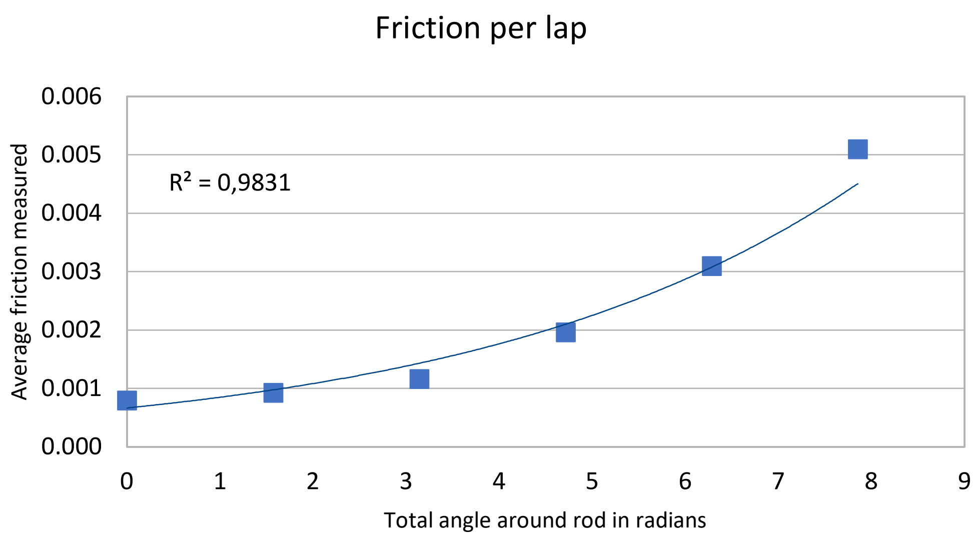
Regarding hypothesis V):

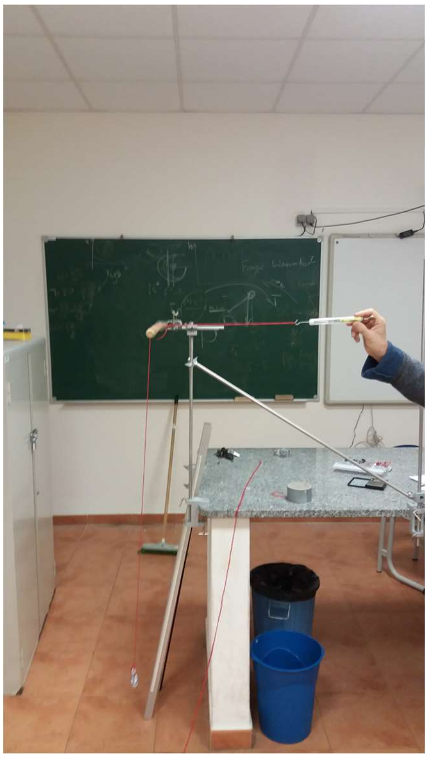
Together with the centripetal force of the light weight, the friction works as counterforce to the gravitational force of the heavy weight.
We use a string with low friction and a metal rod and see the usual archimedes spiral (see “video friction“). However, after the rope wraps around the rod, it starts uncoiling and the heavy weight continues falling to the ground. This means that the friction of the string with itself and with the rod after the occurrence of the spiral must be at least as great as the gravitational force F_{G1} of the heavy weight, because in the end there is no centripetal force anymore. Otherwise, the heavy weight falls to the ground (which I interpret as violation of the task description).

Remark: It can be observed in some cases that the heavy weight slides down a small distance after having come to a halt, reaching a final halt, because the centripetal force temporarily decreases more than the friction increases, since the velocity/kinetic energy decreases. This generally only happens at the highest point of the spiral.

We determine the friction of the rope by measuring the differences between the simple gravitational force of a weight attached to a rope and the force needed to hold a weight on a fixed roller dependent on the total angle (measured in radians) made by all the windings of the rope around the rod:

Total angle in radians	Force needed			
	1. Measurement	2. Measurement	3. Measurement	Average
0	0,8	0,8	0,8	0,800
0,5 π	1	0,9	0,9	0,933
π	1,1	1,2	1,2	1,167
1,5 π	1,8	1,9	2,2	1,967
2 π	3,2	3	3,1	3,100
2,5π	5,4	4,9	5	5,100





The friction increases exponentially when the rope wraps around the rod. This fits in with the so-called Euler-Eytelwein equation (references 7 and 8):

$$T_2 = T_1\, e^{\mu\theta/\sin(\beta/2)}$$

During the movement, the friction increases, since more and more rope wraps around the rod. This means that the counterforce F_f to F_{G1} becomes exponentially larger. I have performed measurements (Friction per lap) for two kinds of strings. It remains to measure the influence of the different frictions on the drop height and spiral shape (→ Work until GYPT). For larger frictions, I expect to see smaller drop heights, since F_f is a counterforce to F_{G1}. L-H-R is then larger, so the spiral radius should be too.

Hypothesis VI):

To prove that rigidity/stiffness plays an important role in the phenomenon, we use a wire as our rope, which obviously has a high rigidity compared with a rope. The “video rigidity 1“ shows that the experiment fails with the wire, although it worked with a rope (control measurement). Because of the increased rigidity, the rope is unable to wrap around the rod sufficiently fast. The „video rigidity 2“ however shows that the rigidity can (with a larger rod, all else equal) in fact also keep the weight from falling since it prevents the wire from unwrapping. Note that the wire and the rope also have different frictions but that hardly explains the failure of the experiment completely. It is very difficult to vary the rigidity without varying the friction.

Conclusion:

This report has shown and aimed to explain, after having set up vaious hypotheses, in which way a number of significant parameters influence the experiment. First, I focused on the drop height of the heavy weight and have for instance shown, that the drop height is smallest when the light weight is released when the string is parallel to the ground/90° to the heavy weight‘s falling direction. I have also shown that a higher weight ratio will increase the drop height.
After having been told about the programme Tracker by Florian Ostermaier, I stopped these measurements and proceeded to record the shape of the spiral, which allows for measurements of velocity, acceleration, etc.
With this different approach, I have already shown that a smaller radius of the rod results in significantly more laps of the light load around the rod.
With three videos I have shown that rigidity and friction play a role in the phenomenon.

Planned Work until GYPT final:

1) More elaborated measurements of all mentioned parameters - using the programme Tracker for all measurements (sadly I only learnt of it quite late) to compare the shapes of the spirals, not only the drop heights and to also perform measurements of velocity and acceleration. Especially, measurements of friction‘s influence on the spiral – until now, I have only done preliminary measurements for friction.

2) A question not yet addressed sufficiently is how much time each lap takes. I want to research this as well. My first hypothesis was that the time per lap decreases from lap to lap, since the length of each lap decreases. However, the measurements with Tracker have shown (figure with measurements) that there is at first a quite constant period that only decreases later (losses due to friction maybe?). Another idea is that the time per lap may stand in relation with the oscillation period of a mathematical pendulum (see background research).

3) I want to analyse the movement of the heavy weight (acceleration and velocity) in detail.

4) Further development of the theoretical model in order to be able to predict the shape of the Archimedean spiral for given parameters. Integration of friction (changing over time) in the formulas. Also, further comparison of theory and experiments, particularly to explain the observations for different angles of deflection.

Video links:

video friction	removed, because anonymised [Youtube link]
video rigidity 1	xxx
video rigidity 2	xxx

References:

1) https://www.youtube.com/watch?v=SXQ9VaYm3yQ

2) https://de.wikipedia.org/wiki/Mathematisches_Pendel

3) http://arithmeticgeometricmean.blogspot.com/

4) https://en.wikipedia.org/wiki/Pendulum

5) https://en.wikipedia.org/wiki/Pendulum_(mathematics)

6) https://en.wikipedia.org/wiki/Centripetal_force

7) https://sciencedemonstrations.fas.harvard.edu/presentations/rope-friction-around-pole

8) https://en.wikipedia.org/wiki/Capstan_equation

9) https://www.stevespanglerscience.com/lab/experiments/magic-pendulum/

10) https://www.istitutotrento5.it/images/test/bre_15_16_looping_pendulum_2_bil.pdf

11) https://www.youtube.com/watch?v=SXQ9VaYm3yQ

12) https://www.youtube.com/watch?v=ZyhHidThQR8

13) https://www.youtube.com/watch?v=XSFXzL4vCPg

14) https://www.mathcurve.com/courbes2d.gb/archimede/archimede.shtml

15) https://rechneronline.de/pi/spiral.php

16) https://www.desmos.com/calculator/4j41rzdkxb